Institute of Actuaries of India

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Mathematics

1. Let f be a function from a set A to A such that $f(f(x)) = x$ for all $x \in A$. Then

   A. f is one-to-one but need not be onto.
   B. f is onto but need not be one-to-one.
   C. f is both one-to-one and onto.
   D. f is neither one-to-one nor onto. 1 mark

2. Let, for real x, $f(x) = 2x + 1$ and $g(x) = \frac{x-1}{2}$. Then

   A. $f \circ g(x) = x$.
   B. $f \circ g(x) = 2x + 1$.
   C. $g \circ f(x) = \frac{x-1}{2}$.
   D. $g \circ f(x) = \frac{(x-1)(2x+1)}{2}$. 1 mark

3. A positive root of the equation $x^3 - 4x - 7 = 0$ lies between

   A. 0 and 1.
   B. 1 and 2.
   C. 2 and 3.
   D. 3 and 4. 1 mark

4. The Maclaurin's series for $\tan^{-1}x$ is

   A. $x + \frac{1}{3}x^3 + \frac{1}{5}x^5 + \cdots$.
   B. $x + \frac{1}{2}x^2 + \frac{1}{4}x^4 + \cdots$.
   C. $x - \frac{1}{2}x^2 + \frac{1}{4}x^4 + \cdots$.
   D. $x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \cdots$. 3 marks

5. If $\binom{n}{k} = 10$, $\binom{n}{k-1} = 5$ and $\binom{n}{k+1} = 10$, then $\binom{n}{k+2}$ is

   A. 1.
   B. 5.
   C. 10.
   D. 15. 2 marks

6. If $\frac{3x+5}{x^2(x-1)} = \frac{A}{x^2} + \frac{B}{x} + \frac{C}{x-1}$, then c^A is equal to

A. $\frac{1}{8^5}$.

B. 8^5.

C. -8^5.

D. $-\frac{1}{8^5}$. 2 marks

7. The sum to n terms of the series $1.6 + 2.7 + 3.8 + \cdots + n(n+5)$ is

A. $\frac{n(n+1)(n+16)}{6}$.

B. $\frac{n(n+1)(2n+1)}{6}$.

C. $\frac{n(n+1)(n+8)}{6}$.

D. $\frac{n(n+1)(n+8)}{3}$. 1 mark

8. The arithmetic mean and geometric mean of the roots of a quadratic equation are a and b, respectively. The quadratic equation is

A. $x^2 - 2ax + b^2 = 0$.
B. $x^2 + 2ax + b^2 = 0$.
C. $x^2 - ax + b^2 = 0$.
D. $x^2 - 2ax - b^2 = 0$. 1 mark

9. $\frac{1}{\log_2 n} + \frac{1}{\log_3 n} + \cdots + \frac{1}{\log_{25} n}$ is

A. $\frac{1}{\log_{25} n}$.

B. $\frac{1}{\log_{25!} n}$.

C. $\log_{25} n$.

D. $\log_{25!} n$. 1 mark

10. $\lim\limits_{x \to 0} \frac{x - \sin x}{x \sin x}$

A. exists and is equal to 0.
B. exists and is equal to 1.
C. exists and is equal to $\frac{1}{2}$.
D. does not exist. 2 marks

11. If the position vectors of A and B are $3\vec{\imath} - 7\vec{\jmath} - 7\vec{k}$ and $5\vec{\imath} + 4\vec{\jmath} + 3\vec{k}$ then the direction cosines of $\overrightarrow{AB}$ are

A. $\left(\frac{2}{225}, \frac{11}{225}, \frac{10}{225}\right)$.

B. $\left(-\frac{2}{15}, -\frac{11}{15}, -\frac{10}{15}\right)$.

C. $\left(\frac{2}{15}, \frac{11}{15}, \frac{10}{15}\right)$.

D. $\left(-\frac{2}{225}, -\frac{11}{225}, -\frac{10}{225}\right)$. 1 mark

12. The cosine of the angle between the vectors $3\vec{\imath} + \vec{\jmath} + 4\vec{k}$ and $\vec{\imath} - \vec{\jmath} + \vec{k}$ is

A. $-\sqrt{\frac{6}{13}}$.

B. $\frac{8}{\sqrt{78}}$.

C. $\frac{6}{\sqrt{26}}$.

D. $\sqrt{\frac{6}{13}}$. 1 mark

13. If $A = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$ and I is the identity matrix of order 2 , then the matrix $AB + BA + I$ is

A. $\begin{bmatrix} 3 & -1 \\ 19 & 5 \end{bmatrix}$.

B. $\begin{bmatrix} 3 & -2 \\ 18 & 5 \end{bmatrix}$.

C. $\begin{bmatrix} 2 & -2 \\ 18 & 4 \end{bmatrix}$.

D. $\begin{bmatrix} 2 & -1 \\ 19 & 5 \end{bmatrix}$. 1 mark

14. If the matrix $\begin{bmatrix} x-1 & 2 & 0 \\ 2 & x-4 & 0 \\ 0 & 0 & x-3 \end{bmatrix}$ is singular, then the solutions for x are

A. $0, 3, 5$.

B. $1, 3, 4$.

C. $0, 3, 4$.

D. $0, 1, 4$. 2 marks

15. The rank of the matrix $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix}$ is

A. 0.

B. 1.

C. 2.

D. 3. 1 mark

16. If the first derivative of $y = (kx - 5)e^{4x}$ at $x = 0$ is -11, then the value of k is

A. -11.
B. 9.
C. 14 .
D. $\frac{1}{11}$. 1 mark

17. If $x = a\sin\theta$ and $y = b\cos\theta$ then $\frac{d^2y}{dx^2}$ is

A. $-\frac{b}{a^2}\sec^2\theta$.
B. $-\frac{b}{a}\sec^2\theta$.
C. $-\frac{b}{a^2}\sec\theta$.
D. $-\frac{b}{a^2}\sec^3\theta$. 3 marks

18. The value of $\int_{-1}^{1}\log_e\left(\frac{2-x}{2+x}\right)dx$ is

A. 0.
B. 1.
C. $\frac{1}{3}$.
D. 3. 1 mark

19. The value of $\int x^3 e^{x^2}dx$ is

A. $e^{x^2}(x^2-1)+c$.
B. $\frac{1}{2}e^{x^2}(x^2-1)+c$.
C. $\frac{1}{2}e^{x^2}(x^2+1)+c$.
D. $e^{x^2}(x^2+1)+c$. 2 marks

20. The values of a function $f(x)$ for some values of x are given in the following table.

x	0	0.2	0.4	0.6	0.8	1.0
$f(x)$	1	0.96154	0.86207	0.73529	0.60976	0.5

Then the approximate value of $\int_0^1 f(x)dx$, using the trapezoid rule, is

A. 0.46687.
B. 0.78373.
C. 1.10060.
D. 7.83732. 2 marks

Statistics

21. Three cards are drawn simultaneously from a box containing six cards numbered $1, 2, 3, 4, 5, 6$. The probability that the selected numbers are in arithmetic progression is

    A. $7/10$.
    B. $3/10$.
    C. $3/5$.
    D. $1/5$. 1 mark

22. If the letters of the word 'ATTENTION' are written down at random, the probability that all the T's occur together is

    A. $1/12$.
    B. $1/72$.
    C. $1/6$.
    D. $7/9$. 1 mark

23. Consider a system consisting of two components C_1 and C_2. It is given that

    $$P(C_1 \text{ fails but } C_2 \text{ does not fail}) = 0.25, \qquad P(C_1 \text{ and } C_2 \text{ fail}) = 0.25.$$

    Then $P(C_2 \text{ fails } | C_1 \text{has failed})$ is

    A. $1/3$.
    B. 1.
    C. $1/2$.
    D. $1/4$. 1 mark

24. Let A and B be two events such that $P(A) = 0.6$, $P(B) = 0.2$ and $P(A|B) = 0.5$. Then $P(\bar{A}\,|\bar{B})$ equals

    A. $1/10$.
    B. $3/10$.
    C. $6/7$.
    D. $3/8$. 1 mark

25. A car manufacturer has two plants. Plant I manufactures 60% of the cars and plant II manufactures 40%. At plant I, 90% of the cars are rated as of standard quality and, at plant II, 80% of the cars are rated as of standard quality. A car is chosen at random and is found to be of standard quality. What is the probability that it has come from plant I?

    A. $27/43$.
    B. $2/3$.
    C. $3/43$.
    D. $16/43$. 2 marks

26. The sum of deviations of 20 observations measured from 30 is -20. The mean of the observations is

A. 20.
B. 29.
C. 30.
D. 21. 1 mark

27. The average marks obtained in an examination by two groups of students was found to be 68 and 73, respectively. The average marks of all the students was 70. The percentage of all students belonging to group 1 is

A. 50.
B. 45.
C. 55.
D. 60. 1 mark

28. The numbers $-1, 0, 3, x, x+2, 9, 12, 13$ are in ascending order. If the median of the numbers is 6, the arithmetic mean of the numbers is

A. 5.
B. 5.5.
C. 6.
D. 6.5. 1 mark

29. The standard deviation and coefficient of variation of a set of observations are 5.2 and 10.4%, respectively. If each observation is increased by 2, then the coefficient of variation of new observations is

A. 10%.
B. 20%.
C. 12.4%.
D. 10.4%. 1 mark

30. Suppose X follows uniform distribution with the probability density function

$$f(x) = \begin{cases} \frac{1}{20}, & 10 \le x \le 30, \\ 0, & \text{otherwise.} \end{cases}$$

Then the interquartile range of the distribution is

A. 5.
B. 10.
C. 15.
D. 20. 1 mark

31. A random variable takes the values 0, 5, 10, 15 and its mean is 10. If $P(X = 15) = 2P(X = 5)$ and $P(X = 10) = 0.3$. Then $P(X = 0)$ is

A. 0.1.
B. 0.2.
C. 0.25.
D. 0.3. 1 mark

32. Let X be a random variable with $E(X) = 5$ and $E(X^2) = 25$. Then $E\big(X + E(X)\big)^3$ is

A. 0.
B. 125.
C. 1000.
D. 250. 3 marks

33. Let X be a non-negative continuous random variable with $P(X > x) = e^{-0.5x}$, $x > 0$. Then $E(X)$ is

A. 0.
B. e^{-1}.
C. 10.
D. 2. 1 mark

34. The number of car accidents per day in a city follows Poisson distribution. The probability that no car accident occurs in a day is 0.1. Then the probability of occurrence of at least two car accidents in a day is

A. $0.9 + 0.1 \times \ln 0.1$.
B. 0.9.
C. $0.9 - \ln 0.1$.
D. $0.9 - 0.1 \times \ln 0.1$. 2 marks

35. Suppose X follows binomial distribution with mean and variance 4 and 2, respectively. Then $P(X \geq 1)$ equals

A. 7/16.
B. 15/16.
C. 247/256.
D. 255/256. 2 marks

36. Suppose the random variable X has a probability density function

$$f(x) = \begin{cases} \frac{|x|}{4}, & -c \le x \le c \\ 0, & \text{otherwise} \end{cases}$$

The value of c is

A. 0.5.
B. 1.
C. 2.
D. 4. 2 marks

37. Let X follow a normal distribution with mean 5 and variance 1.5. Then the mean of $Y = 2X^2 + 3$ is

A. 16.
B. 6.
C. 57.6.
D. 56. 2 marks

38. The joint probability distribution of X and Y is given in the following table.

	y		
x	-1	0	1
0	0.10	0.30	0.20
1	0.05	0.15	0.20

Then $P(Y < X)$ equals

A. 0.7.
B. 0.45.
C. 0.3.
D. 0.2. 1 mark

39. Two random variables X and Y are positively correlated with respective variances 4 and 9. Then the value of k for which $X + k\,Y$ and $X + \frac{2}{3}Y$ are uncorrelated is

A. 2.
B. $-2/3$.
C. 3.
D. $-3/2$. 3 marks

40. The least square regression line of y on x is $x + 5y + 3 = 0$. The least square regression line of $w = 2y + 3$ on $z = x + 2$ is

A. $-2z + 5w + 13 = 0$.
B. $2z + 5w - 13 = 0$.
C. $10z + w - 17 = 0$.
D. $10z + w + 17 = 0$. 2 marks

Data Interpretation and Data Visualization

Answer the questions 41-43 based on the following data.

An organization has 200 employees. The range of salary is Rs. 10000 – 50000. Suppose 30%, 50%, 70% and 85% of the employees receive less than Rs. 15000, Rs. 18000, Rs. 25000, Rs. 30000, respectively, and 5% receive Rs. 40000 and above.

41. The number of employees who receive salary Rs. 30000 and above is

    A. 30.
    B. 25.
    C. 20.
    D. 15. 1 mark

42. The percentage of employees, whose salary is Rs. 15000 or above but less than Rs. 30000, is

    A. 35.
    B. 40.
    C. 50.
    D. 55. 1 mark

43. The ratio of the numbers of employees having salary less than Rs. 15000 to those having salary Rs. 30000 or above is

    A. 1: 2.
    B. 2: 1.
    C. 3: 1.
    D. 3: 2. 1 mark

Answer questions 44 and 45 based on the following data.

The manager of a supermarket in a town gathered the following information on the number of times a customer visits the store during a month. The responses of 40 customers were given below.

3	5	3	1	4	4	5	6	4	2
6	6	6	7	5	4	3	2	4	4
3	5	3	4	5	6	8	4	3	8
4	7	6	5	8	7	6	5	3	2

44. The mode of the distribution of the number of visits is

A. 3.
B. 4.
C. 5.
D. 6.

1 mark

45. The proportion of customers who visit supermarket 5 or more days is

A. 0.15.
B. 0.32.
C. 0.4.
D. 0.5.

1 mark

Answer questions 46 to 48 based on the data given in the following table.

Frequency distribution of workers in a community by age, gender and type of work

Age in Years	Type of Work			
	Manual		Non-manual	
	Male	Female	Male	Female
10 - 15	15	30	0	0
16 - 20	22	25	30	5
21 - 30	35	50	70	40
31 - 45	30	45	45	20
46 - 60	20	15	20	10
above 60	8	5	5	0

46. The share of males in the overall work-force of the community is about

A. 43%.
B. 52%.
C. 55%.
D. 76%. 2 marks

47. The average age of female non-manual workers in the age group 31-60 years is about

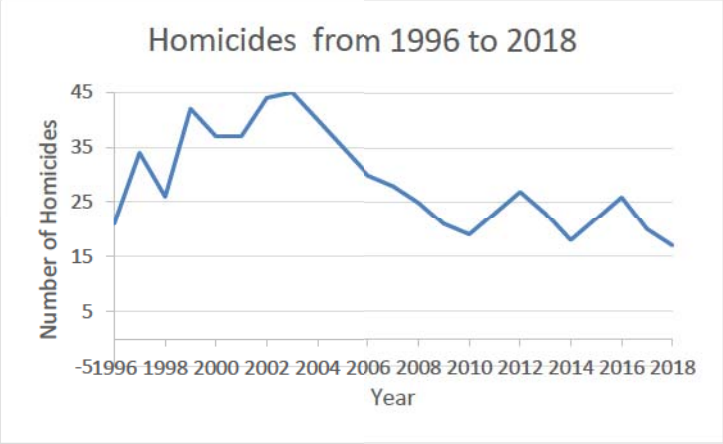
A. 42 years.
B. 42.5 years.
C. 43 years.
D. 46.5 years. 2 marks

48. The highest percentage of females among the manual workers lies in the age group

A. 10-15 years.
B. 16-20 years.
C. 21-30 years.
D. 31-45 years. 2 marks

Answer questions 49 and 50 based on the information given in the following line chart.

Number of homicides from 1996 to 2018 in a province of a country



49. The maximum number of homicides occurred in the year

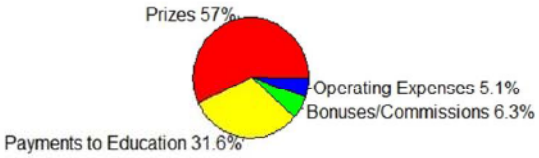
A. 1999.
B. 2002.
C. 2003.
D. 2004. 1 mark

50. The number of years in which the number of homicides was less than that in the year 1996 is

A. 3.
B. 4.
C. 5.
D. 6. 1 mark

Answer question 51 based on the following pie chart.

Expenses of a State Lottery for 2000



51. If the total expenses for the year was Rs. 201.32 crore, then the expenses on the components other than prizes is approximately

A. 83 crores.
B. 87 crores.
C. 88 crores.
D. 90 crores. 2 marks

English

52. Select the word which is not an antonym of "Reprimand":

A. Compliment.
B. Praise.
C. Reward.
D. Berate. 1 mark

53. Select the word which is an antonym of "Revert":

A. Accrue.
B. Progress.
C. Deteriorate.
D. Recur. 1 mark

54. Select the word which is not a synonym of "Capacious":

A. Substantial
B. Cramped
C. Large
D. Significant 1 mark

55. Select the word which is not a synonym of "Recession":

A. Depression
B. Downturn
C. Slump
D. Recovery 1 mark

56. Select the most appropriate word to fill the blank in "The last date ____ submission of the project report is approaching fast":

A. for
B. on
C. to
D. in 1 mark

57. Select the most appropriate word to fill the blank in "He asked her ___ to dinner":

A. up
B. out
C. around
D. in 1 mark

58. One who can do anything for money is called:

A. Mercenary
B. Gullible
C. Infallible
D. Misanthrope 1 mark

59. Select the pair wherein the words have the closest relationship to that of the given pair.

Communication : Message

A. Humour : Amusement
B. Expression : Words
C. Clarification : Doubt
D. Radiation : Cosmic 1 mark

60. Choose from among the alternatives given, the one which will be a substitute for the italicized and underlined expressions in the sentence.

The young sailor who was abandoned on *an uninhabited* island was saved by *timely* arrival of a merchant ship.

A. a rusticated; up-to-date.
B. a marooned; prompt.
C. a bludgeoned; punctual.
D. a deserted; well-timed. 2 marks

61. Select the correct sentence:

A. The complex houses for businessmen and servicemen and their families.
B. The complex houses have been for businessmen and servicemen and their families.
C. The complex houses businessmen and servicemen and their families.
D. The complex houses have for businessmen and servicemen and their families

2 marks

Read the passage below and answer Question No. 62.

The air quality in National Capital Region, Delhi, is the worst among the major cities of the world. Air pollution in India is estimated to kill about 1.5 million people every year. It is the fifth largest cause of death in India. In Delhi, poor quality air irreversibly damages the lungs of 2.2 million or 50 percent of all children. India's Ministry of Earth Sciences published a research paper in October 2018 attributing almost 41% of respirable particulate matter in air at Delhi to vehicular emissions, 21.5% to dust and 18% to industries. Air quality index of Delhi is generally at Moderate (101-200) level between January and September, and then it drastically deteriorates to Very Poor (301-400), Severe (401-500) or Hazardous (500+) levels from October to December, due to various factors including stubble burning, road dust, vehicle pollution and cold weather. In November 2017, in an event known as the Great smog of Delhi, the air pollution spiked far beyond acceptable levels. Levels of PM2.5 and PM10 particulate matter went beyond the instrument limit of 999 micrograms per cubic meter, while the safe limits for those pollutants are 60 and 100, respectively.

I. The air quality of Delhi is
   (i) poorer in September than in October.
   (ii) poorer in October than in September.
   (iii) very poor in January.

II. According to the Ministry of Earth Sciences, the biggest contributor to Delhi's air pollution is
   (i) dust.
   (ii) vehicular emissions.
   (iii) stubble burning.

III. During November 2017 in Delhi, the levels of PM2.5 and PM10
   (i) were far below the respective safe limits.
   (ii) were near the respective safe limits.
   (iii) were far above the respective safe limits.

62. The correct answers to I, II and III are:
   A. iii, iii, iii, respectively.
   B. i, ii, i, respectively.
   C. i, ii, iii, respectively.
   D. ii, ii, iii, respectively. 3 marks

Logical Reasoning

63. In a town of 500 people, 185 read *Hindustan Times*, 212 read *Dainik Jagran*, 127 read *Amar Ujala*, 20 read *Hindustan Times* and *Amar Ujala*, 29 read *Hindustan Times* and *Dainik Jagran*, 35 read *Amar Ujala* and *Dainik Jagran* , and 50 read no newspaper. Then how many read only one newspaper?

    A. 256.
    B. 356.
    C. 326.
    D. 386. 2 marks

64. Find the angle between the hour hand and the minute hand of a clock when the time is 3:25.

    A. 47.5 degrees.
    B. 57.5 degrees.
    C. 45.5 degrees.
    D. 55.5 degrees. 2 marks

65. Four friends U, V, W and X are sitting in a row, facing North. Two extreme seats are occupied by U and X, and V is sitting second to the left of X. Who is sitting to the immediate right of V?

    A. X.
    B. U.
    C. W.
    D. None. 1 mark

66. The second day of a month, which has 30 days is Sunday. What will be the last day of the next month?

    A. Friday.
    B. Saturday.
    C. Monday.
    D. Wednesday. 1 mark

67. Two dice with sides numbered from 1 to 6 are thrown together. The sum of the numbers in the two top faces is 9, and their minimum is an even number. What is the product of these two numbers?

    A. 20.
    B. 18.
    C. 16.
    D. 12. 1 mark

68. There are two types of inhabitants in an island – A type and B type. The A type of inhabitants always speak the truth and the B type inhabitants always lie.

Suresh says, "According to Ramesh, I always speak to truth".

Which of the following is the correct conclusion?

A. Suresh has to be of type A.
B. Suresh has to be of Type B.
C. Ramesh has to be of type A.
D. Ramesh has to be of type B. 1 mark

69. A pyramid of identical oranges has one orange at the top and 25 oranges arranged in a square at the bottom layer. The total number of oranges in the pyramid is

A. 51.
B. 39.
C. 55.
D. 54. 1 mark

70. In the following, two statements numbered as 1 and 2 are given, followed by two conclusions, numbered as I and II. You have to take the two statements to be true even if they seem to be at variance from commonly known facts. Read the conclusions and then decide which of the given conclusions logically follows the given two statements, disregarding the known facts.

Statements

1. Some boys are students.
2. All students are engineers.

Conclusions

I. All engineers are students.
II. Some boys are engineers.

A. Only conclusion I follows.
B. Only Conclusion II follows.
C. Both conclusions I and II follow.
D. Neither conclusion I nor II follows. 1 mark